

# Freedman-Townsend model of topologically massive non-Abelian theory: superfield formalism

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**Abstract:** We perform the Becchi-Rouet-Stora-Tyutin (BRST) analysis of the Freedman-Townsend (FT) model of topologically massive non-Abelian theory by exploiting its (1-form) Yang-Mills (YM) gauge transformations to show the existence of some novel features that are totally different from the results obtained in such a kind of consideration carried out for the dynamical non-Abelian 2-form theory. We tap here the potential of the “augmented” Bonora-Tonin’s superfield approach to BRST formalism to derive the off-shell nilpotent and absolutely anticommuting (anti-)BRST transformations where, in addition to the horizontality condition (HC), we are theoretically compelled to exploit the appropriate gauge-invariant restrictions (GIRs) on the (super)fields for the derivation of symmetries of *all* the relevant fields. We compare our key results with that of the other such attempt for the discussion of the present model within the framework of BRST formalism.

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## 1. Introduction

During the last few years, there has been a great deal of interest in the understanding of the higher  $p$ -form ( $p = 2, 3, 4, \dots$ ) gauge theories because of their relevance in the context of modern developments in (super)string theories and related extended objects (see, e.g. [1-3]). In particular, the merging of 2-form and 1-form (non-)Abelian fields, through the celebrated  $B \wedge F$  term, has led to the development of topologically massive gauge models which provide an alternative to the Higgs mechanism of standard model of particle physics. To be specific, the (non-)Abelian 1-form gauge field acquires a mass in the topologically massive gauge theories (TMGTs) without presence of any Higgs scalar fields [4-9]. In view of the fact that the Higgs particles have not yet been observed experimentally, the above topologically massive models have generated a renewed interest for their study and understanding.

In the context of the above TMGTs, it is pertinent to point out that we have studied the Abelian version [10] of the TMGTs within the framework of the BRST formalism and have also applied the superfield approach to this system. One of the novel observations, in this realm of investigations, has been the emergence and existence of the Curci-Ferrari (CF) type restriction even in the case of this Abelian theory<sup>1</sup>. After applications of the geometrical superfield approaches (see, e.g. [12-15]) to various  $p$ -form gauge theories, we have claimed that one of the essential features of a gauge theory, described within the framework of the BRST formalism, is the existence of the CF type restrictions. We have also shown their deep connections with the geometrical object called gerbes in our systematic study of the (non-)Abelian 1-form and Abelian 2-form and 3-form gauge theories [16,17].

In particular, we have applied the Bonora-Tonin (BT) superfield approach [12,13] to the 4D dynamical non-Abelian 2-form gauge theory (which happens to be a version of the non-Abelian TMGTs) and obtained the off-shell nilpotent and absolutely anticommuting (anti-)BRST symmetry transformations corresponding to its (1-form) Yang-Mills (YM) gauge symmetries as well as its (2-form) tensor gauge symmetries [18]. In a recent couple of papers [19,20], we have shown the existence of some novel features in the BRST analysis of the above dynamical non-Abelian 2-form theory. To be specific, it has been shown that the nilpotent and conserved (anti-)BRST charges, corresponding to the above YM symmetries as well as tensor gauge symmetries, are *not* capable of generating the (anti-)BRST symmetries of some specific fields of the theory. It has been also demonstrated in these works [19,20]

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<sup>1</sup>It should be recalled that the CF condition [11] appeared, for the first time, in the context of the BRST formulation of the 4D non-Abelian 1-form gauge theory.

that, even the requirements of the nilpotency and absolute anticommutativity properties of the (anti-)BRST symmetry transformations, are *not* able to generate the (anti-)BRST transformations of the above cited specific fields.

In our present investigation, we concentrate on the study of the 4D non-Abelian version of the TMGT, proposed by Freedman and Townsend (FT) [4], within the framework of the BT superfield formalism [12,13] to derive the off-shell nilpotent and absolutely anticommuting (anti-)BRST symmetry transformations corresponding to the usual (1-form) YM gauge symmetry transformations of this theory. We demonstrate that the conserved and nilpotent (anti-)BRST charges of our present analysis are *not* able to generate the (anti-)BRST symmetry transformations of the 2-form field  $B_{\mu\nu}$  (which happens to be an auxiliary field in the FT model of non-Abelian TMGT). This is a novel observation *vis-à-vis* our earlier study of the dynamical non-Abelian 2-form gauge theory [19,20]. We lay emphasis on the fact that it is the “augmented” version of the BT superfield formalism, of our present investigation, that is capable of obtaining the (anti-)BRST symmetry transformations for the above auxiliary field ( $B_{\mu\nu}$ ) but the standard generators (i.e. (anti-)BRST charges) as well as the sacrosanct requirements of the nilpotency and anticommutativity are *unable* to generate the above symmetries for the  $B_{\mu\nu}$  field of the FT model (of the non-Abelian version of TMGT).

The main motivating factors behind our present investigation are as follows. First and foremost, the topologically massive 4D (non-)Abelian gauge theories are interesting in their own right as they provide an alternative to the celebrated Higgs mechanism for the generation of mass of the (non-)Abelian 1-form gauge fields. Second, it is very important to perform a comparative study of the existing FT model [4] and dynamical 2-form [6,7,9] topologically massive non-Abelian gauge model so that their relative strengths and weaknesses could become clear. Finally, the problem of constructing a renormalizable, consistent and unitary non-Abelian 2-form gauge theory is still an open challenge. Thus, any deeper understanding of the existing models [4,6], under any systematic scheme, is a welcome step in the direction of achieving our goal of constructing this theory in its full generality.

Our present paper is organized as follows. In Sec. 2, we briefly recapitulate the bare essentials of the (1-form) YM and (2-form) tensor gauge symmetry transformations in the Lagrangian formulation. Our Sec. 3 deals with the derivation of the proper (anti-)BRST symmetry transformations associated with *all* the physical fields and corresponding (anti-)ghost fields (along with the Curci-Ferrari condition) within the framework of “augmented” BT superfield approach to BRST formalism. Sec. 4 and Sec. 5 are devoted to the derivation of (anti-)BRST charges from the (anti-)BRST symmetries of the coupled Lagrangian densities. The conserved ghost charge and BRST

algebra are deduced in Sec. 6. Finally, we summarize our key results, comment on some subtle issues and make some concluding remarks in Sec. 7.

## 2. Preliminaries: continuous gauge symmetries

Let us begin with the following four  $(3 + 1)$ -dimensional (4D) Lagrangian density, proposed by Freedman-Townsend (hereafter called FT model), for the topologically massive non-Abelian gauge theory<sup>2</sup> where there is an explicit coupling between the 1-form non-Abelian gauge field, an additional 1-form non-Abelian field and the 2-form non-Abelian auxiliary field [5], viz;

$$\mathcal{L}_0 = -\frac{1}{4}F^{\mu\nu} \cdot F_{\mu\nu} + \frac{m^2}{2}\Phi^\mu \cdot \Phi_\mu + \frac{m}{4}\varepsilon^{\mu\nu\eta\kappa}\mathcal{F}_{\mu\nu} \cdot B_{\eta\kappa}. \quad (1)$$

Here the 2-form  $F^{(2)} = dA^{(1)} + iA^{(1)} \wedge A^{(1)} \equiv \frac{1}{2!}(dx^\mu \wedge dx^\nu)F_{\mu\nu} \cdot T$  defines the curvature tensor  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - (A_\mu \times A_\nu)$  for the 1-form  $(A^{(1)} = dx^\mu A_\mu \cdot T)$  non-Abelian gauge field  $A_\mu$  and  $\mathcal{F}_{\mu\nu} = F_{\mu\nu} + f_{\mu\nu} - (A_\mu \times \Phi_\nu) - (\Phi_\mu \times A_\nu)$  is the field strength tensor for the sum of potentials  $A_\mu$  and  $\Phi_\mu$ . As a consequence, we note that  $f_{\mu\nu} = \partial_\mu \Phi_\nu - \partial_\nu \Phi_\mu - (\Phi_\mu \times \Phi_\nu)$ . It is self-evident that the 2-form  $[B^{(2)} = \frac{1}{2!}(dx^\mu \wedge dx^\nu)B_{\mu\nu} \cdot T]$  field  $B_{\mu\nu}$  is an auxiliary field in the theory because it does not have a kinetic term. We take note of the fact that  $(A_\mu, B_{\mu\nu}, \Phi_\mu)$  have mass dimension one (in natural units) implying that  $m$ , present in the last term of (1), has the dimension of mass in 4D.

It is straightforward to check that the above Lagrangian density respects  $(\delta_g \mathcal{L}_0 = 0)$  the following (1-form) YM gauge symmetry transformations  $(\delta_g)$

$$\begin{aligned} \delta_g A_\mu &= D_\mu \Omega, & \delta_g B_{\mu\nu} &= -(B_{\mu\nu} \times \Omega), & \delta_g \Phi_\mu &= -(\Phi_\mu \times \Omega), \\ \delta_g F_{\mu\nu} &= -(F_{\mu\nu} \times \Omega), & \delta_g \mathcal{F}_{\mu\nu} &= -(\mathcal{F}_{\mu\nu} \times \Omega), \end{aligned} \quad (2)$$

where  $\Omega = \Omega \cdot T$  is the Lie-valued infinitesimal (Lorentz scalar) gauge parameter and  $D_\mu \Omega = \partial_\mu \Omega - (A_\mu \times \Omega)$  is the covariant derivative w.r.t. the usual 1-form gauge field  $A_\mu$ . We note that the antisymmetric tensor  $f_{\mu\nu}$  does not transform covariantly under the (1-form) YM transformations  $\delta_g$ , namely;

$$\delta_g f_{\mu\nu} = -(f_{\mu\nu} \times \Omega) + (\Phi_\mu \times \partial_\nu \Omega) - (\Phi_\nu \times \partial_\mu \Omega), \quad (3)$$

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<sup>2</sup>We take the signature of the flat metric to be  $(+1, -1, -1, -1)$  for the description of the flat background Minkowski spacetime manifold so that  $P_\mu Q^\mu = P_0 Q_0 - P_i Q_i$  where Greek indices  $\mu, \nu, \eta, \dots = 0, 1, 2, 3$  and Latin indices  $i, j, k, \dots = 1, 2, 3$ . The Levi-Cvita tensor is chosen to be  $\varepsilon_{0123} = +1$  which obeys  $\varepsilon^{\mu\nu\eta\kappa}\varepsilon_{\mu\nu\eta\kappa} = -4!$ ,  $\varepsilon^{\mu\nu\eta\kappa}\varepsilon_{\mu\nu\eta\rho} = -3!\delta_\rho^\kappa$ , etc. For the sake of brevity, we adopt the dot and cross products  $R \cdot S = R^a S^a$ ,  $R \times S = f^{abc} R^a S^b T^c$  in the  $SU(N)$  Lie algebraic space where the generators  $T^a$  of the  $SU(N)$  Lie algebra satisfy the commutator  $[T^a, T^b] = if^{abc} T^c$  with  $a, b, c, \dots = 1, 2, 3, \dots, N^2 - 1$ .

which is essential for the covariant transformation  $\delta_g \mathcal{F}_{\mu\nu} = -(\mathcal{F}_{\mu\nu} \times \Omega)$ . We observe that  $\delta_g(\mathcal{F}_{\mu\nu} \cdot B_{\eta\kappa}) = 0$ . As a consequence, all the terms in the Lagrangian density (1) are *individually* gauge invariant quantities.

There exists a local (2-form) tensor gauge symmetry in the theory. These transformations  $(\delta_t)$ , for all the relevant fields of the theory, are<sup>3</sup>

$$\delta_t F_{\mu\nu} = \delta_t f_{\mu\nu} = \delta_t A_\mu = \delta_t \Phi_\mu = \delta_t \mathcal{F}_{\mu\nu} = 0, \quad \delta_t B_{\mu\nu} = \tilde{D}_\mu \Lambda_\nu - \tilde{D}_\nu \Lambda_\mu. \quad (4)$$

Here  $\tilde{D}_\mu \Lambda_\nu = \partial_\mu \Lambda_\nu - (A_\mu \times \Lambda_\nu) - (\Phi_\mu \times \Lambda_\nu)$  is the covariant derivative w.r.t. the sum of 1-form fields  $A_\mu$  and  $\Phi_\mu$ , and  $\Lambda_\mu = \Lambda_\mu \cdot T$  is the infinitesimal Lorentz vector gauge transformation parameter. In fact, the validity of the Bianchi identity  $\tilde{D}_\mu \mathcal{F}_{\nu\eta} + \tilde{D}_\nu \mathcal{F}_{\eta\mu} + \tilde{D}_\eta \mathcal{F}_{\mu\nu} = 0$  is the root cause of the symmetry invariance of the Lagrangian density (and corresponding action integral) under the above local tensor gauge symmetry transformations.

### 3. (Anti-)BRST symmetries: superfield formalism

We discuss here separately the derivation of the nilpotent (anti-)BRST symmetry transformations corresponding to the (1-form) YM gauge transformations (2) for (i) the non-Abelian 1-form gauge field  $A_\mu$  and corresponding (anti-)ghost fields, and (ii) the 2-form non-Abelian field  $B_{\mu\nu}$  and 1-form field  $\Phi_\mu$ . We also comment on the derivation of CF condition and its importance.

#### 3.1 (Anti-)BRST symmetries: 1-form gauge and ghost fields

We briefly recapitulate here the Bonora-Tonin's superfield approach to BRST formalism [12,13] and derive the (anti-)BRST transformations for the 1-form field  $A_\mu$  and (anti-)ghost fields  $(\bar{C})C$  along with the Curci-Ferrari condition [11]. To this end in mind, first of all, we generalize the curvature 2-form, defined in the language of Maurer-Cartan equation  $F^{(2)} = dA^{(1)} + i A^{(1)} \wedge A^{(1)}$  (and its innate operators), onto the (4, 2)-dimensional supermanifold, as [12]

$$\begin{aligned} d &\rightarrow \tilde{d} = dZ^M \partial_M \equiv dx^\mu \partial_\mu + d\theta \partial_\theta + d\bar{\theta} \partial_{\bar{\theta}}, \\ A^{(1)} &\rightarrow \tilde{A}^{(1)} = dZ^M \tilde{A}_M \equiv dx^\mu \tilde{B}_\mu(x, \theta, \bar{\theta}) + d\theta \tilde{\bar{F}}(x, \theta, \bar{\theta}) + d\bar{\theta} \tilde{F}(x, \theta, \bar{\theta}), \\ F^{(2)} &\rightarrow \tilde{F}^{(2)} = \frac{1}{2!} (dZ^M \wedge dZ^N) \tilde{F}_{MN} \equiv \tilde{d} \tilde{A}^{(1)} + i \tilde{A}^{(1)} \wedge \tilde{A}^{(1)}, \end{aligned} \quad (5)$$

where the superspace coordinates  $Z^M = (x^\mu, \theta, \bar{\theta})$  characterize the above (4, 2)-dimensional supermanifold with spacetime variables  $x^\mu$  (with  $\mu =$

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<sup>3</sup>Of course, we shall *not* be exploiting these continuous symmetry transformations for our present discussions on the derivation of (anti-)BRST symmetry transformations within the framework of “augmented” Bonora-Tonin's superfield approach to BRST formalism.

0, 1, 2, 3) and a pair of Grassmannian variables (with  $\theta^2 = \bar{\theta}^2 = 0, \theta\bar{\theta} + \bar{\theta}\theta = 0$ ). Corresponding superspace derivatives are denoted by  $\partial_M = (\partial_\mu, \partial_\theta, \partial_{\bar{\theta}})$ .

The set of superfields  $(\tilde{B}_\mu, \tilde{F}, \tilde{\bar{F}})$  form the super-multiplet of a super vector field  $\tilde{A}_M$  and these have the following super expansions [12]

$$\begin{aligned}\tilde{B}_\mu(x, \theta, \bar{\theta}) &= A_\mu(x) + \theta \bar{R}_\mu(x) + \bar{\theta} R_\mu(x) + i \theta \bar{\theta} S_\mu(x), \\ \tilde{F}(x, \theta, \bar{\theta}) &= C(x) + i \theta \bar{B}_1(x) + i \bar{\theta} B_1(x) + i \theta \bar{\theta} s(x), \\ \tilde{\bar{F}}(x, \theta, \bar{\theta}) &= \bar{C}(x) + i \theta \bar{B}_2(x) + i \bar{\theta} B_2(x) + i \theta \bar{\theta} \bar{s}(x),\end{aligned}\quad (6)$$

in terms of the basic fields  $(A_\mu, C, \bar{C})$  of the BRST invariant non-Abelian 1-form theory [21] and secondary fields  $R_\mu, \bar{R}_\mu, S_\mu, B_1, \bar{B}_1, B_2, \bar{B}_2, s, \bar{s}$ . The secondary fields are determined in terms of the basic and auxiliary fields of the 4D non-Abelian 1-form theory by imposing the celebrated horizontality condition (HC) which states that all the Grassmannian components of the super 2-form  $\tilde{F}^{(2)} = \frac{1}{2!}(dZ^M \wedge dZ^N)\tilde{F}_{MN}$  should be set equal to zero (see, e.g., [12]). The explicit computation of  $\tilde{F}^{(2)}$  and application of HC, lead to

$$\begin{aligned}R_\mu &= D_\mu C, \quad \bar{R}_\mu = D_\mu \bar{C}, \quad B_1 = -\frac{i}{2}(C \times C), \quad \bar{B}_2 = -\frac{i}{2}(\bar{C} \times \bar{C}), \\ S_\mu &= D_\mu B + i(D_\mu C \times \bar{C}) \equiv -D_\mu \bar{B} - i(D_\mu \bar{C} \times C), \\ s &= -(\bar{B} \times C), \quad \bar{s} = +(B \times \bar{C}), \quad B + \bar{B} = -i(C \times \bar{C}),\end{aligned}\quad (7)$$

where we have identified  $B_2 = B, \bar{B}_1 = \bar{B}$  to be consistent with the Nakanishi-Lautrup notations of the auxiliary fields in the 4D non-Abelian 1-form theory. We also note here that we have already derived the celebrated Curci-Ferrari (CF) condition (i.e.  $B + \bar{B} = -i(C \times \bar{C})$ ) for the non-Abelian theory [21].

With the above substitutions, we obtain the following super expansions of the above multiplet superfields<sup>4</sup> (after the application of the HC) [12]

$$\begin{aligned}\tilde{B}_\mu^{(h)}(x, \theta, \bar{\theta}) &= A_\mu(x) + \theta \bar{D}_\mu \bar{C}(x) + \bar{\theta} D_\mu C(x) \\ &+ \theta \bar{\theta} [i D_\mu C - (D_\mu C \times \bar{C})](x), \\ &\equiv A_\mu(x) + \theta (s_{ab} A_\mu(x)) + \bar{\theta} (s_b A_\mu(x)) + \theta \bar{\theta} (s_b s_{ab} A_\mu(x)), \\ \tilde{F}^{(h)}(x, \theta, \bar{\theta}) &= C(x) + \theta (i \bar{B}(x)) + \bar{\theta} \left[ \frac{1}{2}(C \times C)(x) \right] + \theta \bar{\theta} [-i(\bar{B} \times C)(x)], \\ &\equiv C(x) + \theta (s_{ab} C(x)) + \bar{\theta} (s_b C(x)) + \theta \bar{\theta} (s_b s_{ab} C(x)), \\ \tilde{\bar{F}}^{(h)}(x, \theta, \bar{\theta}) &= \bar{C}(x) + \theta \left[ \frac{1}{2}(\bar{C} \times \bar{C})(x) \right] + \bar{\theta} (i \bar{B}(x)) + \theta \bar{\theta} [(+i(B \times \bar{C}))(x)], \\ &\equiv \bar{C}(x) + \theta (s_{ab} \bar{C}(x)) + \bar{\theta} (s_b \bar{C}(x)) + \theta \bar{\theta} (s_b s_{ab} \bar{C}(x)),\end{aligned}\quad (8)$$

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<sup>4</sup>The superscript  $(h)$  on the multiplet superfields denotes the fact that the expansions have been obtained after the application of celebrated HC.

which define the (anti-)BRST symmetry transformations  $s_{(a)b}$  for the 1-form non-Abelian gauge field and corresponding (anti-)ghost fields as

$$\begin{aligned} s_b A_\mu &= D_\mu C, \quad s_b C = \frac{1}{2}(C \times C), \quad s_b \bar{B} = -(\bar{B} \times C), \quad s_b B = 0, \\ s_{ab} A_\mu &= D_\mu \bar{C}, \quad s_{ab} \bar{C} = \frac{1}{2}(\bar{C} \times \bar{C}), \quad s_{ab} B = -(B \times C), \quad s_{ab} \bar{B} = 0. \end{aligned} \quad (9)$$

We note, from equation (8), that  $s_b \leftrightarrow \text{Lim}_{\theta \rightarrow 0}(\partial/\partial\bar{\theta})$ ,  $s_{ab} \leftrightarrow \text{Lim}_{\bar{\theta} \rightarrow 0}(\partial/\partial\theta)$ . It is worthwhile to mention that the requirements of the off-shell nilpotency ( $s_{(a)b}^2 = 0$ ) and absolute anticommutativity ( $s_b s_{ab} + s_{ab} s_b = 0$ ) of the above (anti-)BRST symmetry transformations leads to the derivation of the nilpotent (anti-)BRST transformations for the Nakanishi-Lautrup auxiliary fields as:  $s_b B = 0$ ,  $s_b \bar{B} = -(\bar{B} \times C)$ ,  $s_{ab} \bar{B} = 0$ ,  $s_{ab} B = -(B \times \bar{C})$ , etc.

We wrap up this subsection with the following remarks. First, the absolute anticommutativity  $\{s_b, s_{ab}\}A_\mu = 0$  is true if and only if the CF condition  $(B + \bar{B} + i(C \times \bar{C}) = 0)$  is satisfied. Second, it can be checked that CF condition is (anti-)BRST invariant (i.e.  $s_{(a)b}[B + \bar{B} + i(C \times \bar{C})] = 0$ ) quantity and, therefore, this condition is physical (in some sense). Finally, spacetime component of the super 2-form  $\tilde{F}^{(2)}$  leads to the derivation of the following super expansion of the antisymmetric super curvature [12]:

$$\begin{aligned} \tilde{F}_{\mu\nu}^{(h)}(x, \theta, \bar{\theta}) &= F_{\mu\nu} - \theta (F_{\mu\nu} \times \bar{C}) - \bar{\theta} (F_{\mu\nu} \times C) \\ &+ \theta \bar{\theta} [(F_{\mu\nu} \times C) \times \bar{C} - i F_{\mu\nu} \times B], \end{aligned} \quad (10)$$

which implies the following nilpotent and anticommuting (anti-)BRST symmetry transformations for it (i.e. the curvature tensor), namely;

$$\begin{aligned} s_b F_{\mu\nu} &= -(F_{\mu\nu} \times C), \quad s_{ab} F_{\mu\nu} = -(F_{\mu\nu} \times \bar{C}), \\ s_b s_{ab} F_{\mu\nu} &= (F_{\mu\nu} \times C) \times \bar{C} - i F_{\mu\nu} \times B. \end{aligned} \quad (11)$$

We observe that  $-\frac{1}{4}\tilde{F}^{\mu\nu(h)}(x, \theta, \bar{\theta}) \cdot \tilde{F}_{\mu\nu}^{(h)}(x, \theta, \bar{\theta}) = -\frac{1}{4}F^{\mu\nu}(x) \cdot F_{\mu\nu}(x)$ . The Grassmannian independence of the l.h.s. implies that the kinetic term remains invariant under the above (anti-)BRST symmetry transformations in view of the fact that  $s_b \leftrightarrow \text{Lim}_{\theta \rightarrow 0}(\partial/\partial\bar{\theta})$ ,  $s_{ab} \leftrightarrow \text{Lim}_{\bar{\theta} \rightarrow 0}(\partial/\partial\theta)$ .

### 3.2 (Anti-)BRST symmetries: 1-form field $\Phi_\mu$ and 2-form field $B_{\mu\nu}$

As pointed out earlier, it can be checked that there are useful combinations of fields that are gauge-invariant quantities under the 1-form YM transformations (2). To be specific, it can be verified that

$$\delta_g(F_{\mu\nu} \cdot B_{\eta\kappa}) = 0, \quad \delta_g(F_{\mu\nu} \cdot \Phi_\eta) = 0. \quad (12)$$

As a consequence, these quantities are physical objects as far as the gauge transformations (2) are concerned. Thus, we demand that these quantities should remain independent of the Grassmannian variables when they are generalized onto the (4, 2)-dimensional supermanifold. In other words, we invoke the following gauge invariant restrictions (GIRs) on the (super)fields:

$$\begin{aligned}\tilde{F}_{\mu\nu}^{(h)}(x, \theta, \bar{\theta}) \cdot \tilde{B}_{\eta\kappa}(x, \theta, \bar{\theta}) &= F_{\mu\nu}(x) \cdot B_{\eta\kappa}(x), \\ \tilde{F}_{\mu\nu}^{(h)}(x, \theta, \bar{\theta}) \cdot \tilde{\Phi}_\eta(x, \theta, \bar{\theta}) &= F_{\mu\nu}(x) \cdot \Phi_\eta(x).\end{aligned}\quad (13)$$

Physically, the above requirements imply the (anti-)BRST invariance of the gauge invariant quantities listed in (12). The above choice is important because, with the inputs from the super expansion in (10), we shall be able to obtain the (anti-)BRST transformations for the fields  $\Phi_\mu$  and  $B_{\mu\nu}$ .

Towards above goal in mind, let us exploit the following general expansions for the superfields in the GIRs, listed in (13), namely;

$$\begin{aligned}\tilde{B}_{\mu\nu}(x, \theta, \bar{\theta}) &= B_{\mu\nu}(x) + \theta \bar{R}_{\mu\nu}(x) + \bar{\theta} R_{\mu\nu}(x) + i \theta \bar{\theta} S_{\mu\nu}(x), \\ \tilde{\Phi}_\mu(x, \theta, \bar{\theta}) &= \Phi_\mu(x) + \theta \bar{S}_\mu(x) + \bar{\theta} S_\mu(x) + i \theta \bar{\theta} T_\mu(x),\end{aligned}\quad (14)$$

where  $(\Phi_\mu, T_\mu, B_{\mu\nu}, S_{\mu\nu})$  are the bosonic (even) fields and  $(S_\mu, \bar{S}_\mu, R_{\mu\nu}, \bar{R}_{\mu\nu})$  are the fermionic (odd) fields in the above super expansion. The secondary fields  $S_\mu, \bar{S}_\mu, T_\mu, R_{\mu\nu}, \bar{R}_{\mu\nu}, S_{\mu\nu}$  are to be determined from the GIRs given in (13). In fact, with the help of (13) and (14), we have the following

$$\begin{aligned}R_{\mu\nu} &= -(B_{\mu\nu} \times C), \quad \bar{R}_{\mu\nu} = -(B_{\mu\nu} \times \bar{C}), \\ S_{\mu\nu} &= -i[(B_{\mu\nu} \times C) \times \bar{C} - i(B_{\mu\nu} \times B)], \\ S_\mu &= -(\Phi_\mu \times C), \quad \bar{S}_\mu = -(\Phi_\mu \times \bar{C}), \\ T_\mu &= -i[(\Phi_\mu \times C) \times \bar{C} - i(\Phi_\mu \times B)].\end{aligned}\quad (15)$$

The substitution of the above expressions in (14) leads to<sup>5</sup>

$$\begin{aligned}\tilde{B}_{\mu\nu}^{(g)}(x, \theta, \bar{\theta}) &= B_{\mu\nu} - \theta (B_{\mu\nu} \times C)(x) + \bar{\theta} (B_{\mu\nu} \times \bar{C})(x) \\ &\quad + \theta \bar{\theta} [(B_{\mu\nu} \times C) \times \bar{C} - i(B_{\mu\nu} \times B)](x), \\ \tilde{\Phi}_\mu^{(g)}(x, \theta, \bar{\theta}) &= \Phi_\mu(x) - \theta (\Phi_\mu \times \bar{C})(x) + \bar{\theta} (\Phi_\mu \times C)(x) \\ &\quad + \theta \bar{\theta} [(\Phi_\mu \times C) \times \bar{C} - i(\Phi_\mu \times B)](x),\end{aligned}\quad (16)$$

where the superscript  $(g)$  on the superfields denotes that the superfields have been obtained after the application of the GIRs (listed in (13)). Taking the

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<sup>5</sup>It can be checked that  $\tilde{\Phi}^{\mu(g)}(x, \theta, \bar{\theta}) \cdot \tilde{\Phi}_\mu^{(g)}(x, \theta, \bar{\theta}) = \Phi_\mu(x) \cdot \Phi^\mu(x)$  and  $\tilde{B}^{\mu\nu(g)}(x, \theta, \bar{\theta}) \cdot \tilde{B}_{\mu\nu}^{(g)}(x, \theta, \bar{\theta}) = B^{\mu\nu}(x) \cdot B_{\mu\nu}(x)$  which demonstrate the gauge and (anti-)BRST invariance of  $(\Phi^\mu \cdot \Phi_\mu)$  and  $(B^{\mu\nu} \cdot B_{\mu\nu})$  in view of  $s_b \leftrightarrow \text{Lim}_{\theta \rightarrow 0}(\partial/\partial\theta)$ ,  $s_{ab} \leftrightarrow \text{Lim}_{\bar{\theta} \rightarrow 0}(\partial/\partial\bar{\theta})$ .



inputs from our previous discussions, it is clear that the off-shell nilpotent ( $s_{(a)b}^2 = 0$ ) (anti-)BRST transformations for the fields  $\Phi_\mu$  and  $B_{\mu\nu}$  are

$$\begin{aligned} s_b \Phi_\mu &= -(\Phi_\mu \times C), & s_b B_{\mu\nu} &= -(B_{\mu\nu} \times C), \\ s_b s_{ab} \Phi_\mu &= (\Phi_\mu \times C) \times \bar{C} - i (\Phi_\mu \times B), \\ s_{ab} \Phi_\mu &= -(\Phi_\mu \times \bar{C}), & s_{ab} B_{\mu\nu} &= -B_{\mu\nu} \times \bar{C} \\ s_b s_{ab} B_{\mu\nu} &= (B_{\mu\nu} \times C) \times \bar{C} - i B_{\mu\nu} \times B. \end{aligned} \quad (17)$$

Thus, the set of transformations (9) and (17) produce all the off-shell nilpotent (anti-)BRST symmetry transformations for *all* the basic fields of our present 4D topologically massive gauge theory.

We have seen earlier that the curvature tensor  $\mathcal{F}_{\mu\nu}$  transforms covariantly under the (1-form) YM gauge transformations (2) (i.e.  $\delta_g \mathcal{F}_{\mu\nu} = -(\mathcal{F}_{\mu\nu} \times \Omega)$ ). Its transformations under the (anti-)BRST symmetry transformations can also be obtained within the framework of “augmented” BT superfield formalism. This can be obtained by plugging in the explicit expansions of  $\tilde{B}_\mu^{(h)}, \tilde{\Phi}_\mu^{(g)}, \tilde{F}_{\mu\nu}^{(h)}, \tilde{f}_{\mu\nu}^{(g)}$  in the following expression for this super curvature tensor

$$\tilde{\mathcal{F}}_{\mu\nu}^{(g,h)}(x, \theta, \bar{\theta}) = \tilde{F}_{\mu\nu}^{(h)} + \tilde{f}_{\mu\nu}^{(g)} - (\tilde{B}_\mu^{(h)} \times \tilde{\Phi}_\nu^{(g)}) + (\tilde{B}_\nu^{(h)} \times \tilde{\Phi}_\mu^{(g)}), \quad (18)$$

where the explicit expression for  $\tilde{f}_{\mu\nu}^{(g)}$  is as given below

$$\begin{aligned} \tilde{f}_{\mu\nu}^{(g)}(x, \theta, \bar{\theta}) &= \partial_\mu \tilde{\Phi}_\nu^{(g)} - \partial_\nu \tilde{\Phi}_\mu^{(g)} - (\tilde{\Phi}_\mu^{(g)} \times \tilde{\Phi}_\nu^{(g)}) \\ &\equiv f_{\mu\nu}(x) + \theta (s_{ab} f_{\mu\nu}(x)) + \bar{\theta} (s_b f_{\mu\nu}(x)) + \theta \bar{\theta} (s_b s_{ab} f_{\mu\nu}(x)). \end{aligned} \quad (19)$$

In the above, the explicit forms of the nilpotent (anti-)BRST symmetry transformations for the curvature tensor  $f_{\mu\nu}$  are listed below

$$\begin{aligned} s_b f_{\mu\nu} &= -(f_{\mu\nu} \times C) + (\Phi_\mu \times \partial_\nu C) - (\Phi_\nu \times \partial_\mu C), \\ s_{ab} f_{\mu\nu} &= -(f_{\mu\nu} \times \bar{C}) + (\Phi_\mu \times \partial_\nu \bar{C}) - (\Phi_\nu \times \partial_\mu \bar{C}), \\ s_b s_{ab} f_{\mu\nu} &= (f_{\mu\nu} \times C) \times \bar{C} - i (f_{\mu\nu} \times B) \\ &\quad - (\Phi_\mu \times \partial_\nu C) \times \bar{C} + (\Phi_\nu \times C) \times \partial_\mu \bar{C} \\ &\quad + (\Phi_\nu \times \partial_\mu C) \times \bar{C} - (\Phi_\mu \times C) \times \partial_\nu \bar{C} + i (\Phi_\mu - \Phi_\nu) \times B. \end{aligned} \quad (20)$$

The substitutions of the expressions in (10), (16), (19), (20), with a little dose of algebra, leads to the following expansion for the super curvature tensor<sup>6</sup>

$$\begin{aligned} \tilde{\mathcal{F}}_{\mu\nu}^{(g,h)}(x, \theta, \bar{\theta}) &= \mathcal{F}_{\mu\nu}(x) - \theta (\mathcal{F}_{\mu\nu} \times \bar{C})(x) - \bar{\theta} (\mathcal{F}_{\mu\nu} \times C)(x) \\ &\quad + \theta \bar{\theta} [(\mathcal{F}_{\mu\nu} \times C) \times \bar{C} - i (\mathcal{F}_{\mu\nu} \times B)](x). \end{aligned} \quad (21)$$

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<sup>6</sup>It is interesting to check that  $\tilde{\mathcal{F}}_{\mu\nu}^{(g,h)}(x, \theta, \bar{\theta}) \cdot \tilde{B}_{\eta\kappa}^{(g)}(x, \theta, \bar{\theta}) = \mathcal{F}_{\mu\nu}(x) \cdot B_{\eta\kappa}(x)$ . This shows the (anti-)BRST invariance of the topological massive term of the Lagrangian density (1).

The above expansion leads to the derivation of the (anti-)BRST symmetry transformations for the curvature tensor  $\mathcal{F}_{\mu\nu}$  as given below

$$\begin{aligned} s_b \mathcal{F}_{\mu\nu} &= -(\mathcal{F}_{\mu\nu} \times C), & s_{ab} \mathcal{F}_{\mu\nu} &= -(\mathcal{F}_{\mu\nu} \times \bar{C}), \\ s_b s_{ab} \mathcal{F}_{\mu\nu} &= (\mathcal{F}_{\mu\nu} \times C) \times \bar{C} - i (\mathcal{F}_{\mu\nu} \times B). \end{aligned} \quad (22)$$

Thus, we have derived all the proper (anti-)BRST symmetry transformations for all the dynamical fields and their curvature tensors (that are present in the theory) by exploiting the “augmented” BT superfield approach to BRST formalism where the HC and GIRs blend together in a meaningful manner.

### 3.3 Curci-Ferrari condition: absolute anticommutativity of the nilpotent (anti-)BRST symmetries and coupled Lagrangian densities

One of the key signatures of a  $p$ -form ( $p = 1, 2, 3, \dots$ ) gauge theory is the existence of the first-class constraints in the language of Dirac’s prescription for the classification scheme [22,23]. When these theories are discussed within the framework of BRST formalism, there always exists (one or more number of) Curci-Ferrari (CF) type conditions. For instance, in the simplest case of an Abelian 1-form (anti-)BRST invariant gauge theory, we have a trivial CF type condition  $B + \bar{B} = 0$  as can be seen from the Abelian limit of the CF condition (i.e.  $B + \bar{B} + i(C \times \bar{C}) = 0$ ) mentioned in (7).

The beauty of the Bonora-Tonin’s (BT) superfield approach to BRST formalism is that the above CF-type restrictions emerge very naturally<sup>7</sup>. They are always (anti-)BRST invariant as has been proven within the framework of “augmented” BT superfield approach to the non-Abelian TMGT where the dynamical 2-form gauge field is coupled with the 1-form gauge field through the celebrated  $B \wedge F$  term (see, e.g. [18] for details). In fact, the existence of CF type restrictions are responsible for the absolute anticommutativity of the (anti-)BRST symmetry transformations in the context of any arbitrary  $p$ -form gauge theory, discussed within the framework of BRST formalism. For instance, it can be explicitly checked that, in our present topologically massive theory, the following anticommutators

$$\{s_b, s_{ab}\} A_\mu = 0, \quad \{s_b, s_{ab}\} \Phi_\mu = 0, \quad \{s_b, s_{ab}\} B_{\mu\nu} = 0, \quad (23)$$

are true if and only if we take into account the (anti-)BRST invariant CF condition  $B + \bar{B} + i(C \times \bar{C}) = 0$ . This anticommutativity property is valid

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<sup>7</sup>In fact, it turns out that when the coefficient of the differential ( $d\theta \wedge d\bar{\theta}$ ) of the super curvature 2-form  $\tilde{F}^{(2)} = \frac{1}{2!}(dZ^M \wedge dZ^N)\tilde{F}_{MN}$  is set equal to zero due to HC, we obtain the CF condition  $B + \bar{B} + i(C \times \bar{C}) = 0$  within the framework of BT superfield formalism.

for all the *rest* of the fields (of our present 4D topologically massive theory) *without* invoking the above CF type restriction in any form.

Another important contribution of the CF type restrictions is the derivation of the coupled (but equivalent) Lagrangian densities for a given  $p$ -form gauge theory. In the case of a simple Abelian 1-form gauge theory, these coupled Lagrangian densities merge into a single Lagrangian density. For our case, it can be checked that the following Lagrangian densities<sup>8</sup>

$$\begin{aligned}\mathcal{L}_B &= \mathcal{L}_0 + s_b s_{ab} \left[ \frac{i}{2} A_\mu \cdot A^\mu + \frac{1}{4} B_{\mu\nu} \cdot B^{\mu\nu} + \frac{i}{2} \Phi_\mu \cdot \Phi^\mu + \bar{C} \cdot C \right], \\ \mathcal{L}_{\bar{B}} &= \mathcal{L}_0 - s_{ab} s_b \left[ \frac{i}{2} A_\mu \cdot A^\mu + \frac{1}{4} B_{\mu\nu} \cdot B^{\mu\nu} + \frac{i}{2} \Phi_\mu \cdot \Phi^\mu + \bar{C} \cdot C \right],\end{aligned}\quad (24)$$

are found to respect the (anti-)BRST symmetry transformations on a surface in the 4D spacetime manifold where the CF condition  $B + \bar{B} + i(C \times \bar{C}) = 0$  is satisfied. In fact, taking the help of transformations (9) and (17), we can derive the above Lagrangian densities explicitly as given below (see, e.g. [21])

$$\begin{aligned}\mathcal{L}_B &= \mathcal{L}_0 + B \cdot (\partial_\mu A^\mu) + \frac{1}{2} (B \cdot B + \bar{B} \cdot \bar{B}) - i \partial_\mu \bar{C} \cdot D^\mu C, \\ \mathcal{L}_{\bar{B}} &= \mathcal{L}_0 - \bar{B} \cdot (\partial_\mu A^\mu) + \frac{1}{2} (B \cdot B + \bar{B} \cdot \bar{B}) - i D_\mu \bar{C} \cdot \partial^\mu C.\end{aligned}\quad (25)$$

It is worthwhile to mention that the (1-form) YM gauge (and (anti-)BRST) invariance of  $(B^{\mu\nu} \cdot B_{\mu\nu})$  and  $(\Phi^\mu \cdot \Phi_\mu)$  in (24) imply that there are no gauge-fixing and Faddeev-Popov ghost terms for the fields  $B_{\mu\nu}$  and  $\Phi_\mu$  that could be incorporated in (25). The equivalence of the above Lagrangian densities can be easily checked by the following equality

$$B \cdot (\partial_\mu A^\mu) - i \partial_\mu \bar{C} \cdot D^\mu C = -\bar{B} \cdot (\partial_\mu A^\mu) - i D_\mu \bar{C} \cdot \partial^\mu C, \quad (26)$$

which is valid only on a surface in the 4D Minkowski spacetime manifold that is described by the field equation  $B + \bar{B} + i(C \times \bar{C}) = 0$ . In other words, the above (anti-)BRST invariant CF condition is responsible for the derivation of the (anti-)BRST invariant coupled Lagrangian densities (25).

As a closing remark to this subsection, we would like to point out that there is yet another way to derive the CF condition besides the BT method of superfield formalism which produces it very naturally [12]. For instance, one can derive the following Euler-Lagrange equations of motion for the 1-form

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<sup>8</sup>It should be noted that, for the 4D (anti-)BRST invariant theory, the terms inside the square brackets are unique in the sense that these are the only combinations that have mass dimensions two (in natural units) and ghost number equal to zero.

gauge field from the coupled Lagrangian densities given in (25), namely;

$$\begin{aligned} D_\mu F^{\mu\nu} + \frac{m}{2} \varepsilon^{\nu\mu\eta\kappa} \tilde{D}_\mu B_{\eta\kappa} - \partial^\nu B &= +i (\partial^\nu \bar{C} \times C), \\ D_\mu F^{\mu\nu} + \frac{m}{2} \varepsilon^{\nu\mu\eta\kappa} \tilde{D}_\mu B_{\eta\kappa} + \partial^\nu \bar{B} &= -i (\bar{C} \times \partial^\nu C). \end{aligned} \quad (27)$$

If we take the difference of the above equations, we easily obtain the CF condition  $B + \bar{B} + i (C \times \bar{C}) = 0$ . We conclude, ultimately, that the (anti-)BRST invariant CF condition is hidden in the coupled (but equivalent) Lagrangian densities (25) of our present theory, in a subtle manner.

#### 4. BRST charge as the generator: a new feature

Let us focus on the BRST invariant Lagrangian density  $\mathcal{L}_B$  (cf. (25)). It can be checked that this Lagrangian density transforms to a total spacetime derivative (i.e.  $s_b \mathcal{L}_B = \partial_\mu [B \cdot D^\mu C]$ ) under the BRST transformations  $s_b$  (cf. (9),(11),(17),(22)). This symmetry invariance can be captured within the framework of “augmented” BT superfield formalism because, the super Lagrangian density of our present theory (i.e. an analogue of  $\mathcal{L}_B$ ) is given by

$$\begin{aligned} \tilde{\mathcal{L}}_B &= -\frac{1}{4} \tilde{F}^{\mu\nu(h)} \cdot \tilde{F}_{\mu\nu}^{(h)} + \frac{m^2}{2} \tilde{\Phi}^{\mu(g)} \cdot \tilde{\Phi}_\mu^{(g)} + \frac{m}{4} \varepsilon^{\mu\nu\eta\kappa} \tilde{\mathcal{F}}_{\mu\nu}^{(g,h)} \cdot \tilde{B}_{\eta\kappa}^{(g)} \\ &+ \frac{\partial}{\partial \bar{\theta}} \frac{\partial}{\partial \theta} \left[ \frac{i}{2} \tilde{B}^{\mu(h)} \cdot \tilde{B}_\mu^{(h)} + \frac{1}{4} \tilde{B}^{\mu\nu(g)} \cdot \tilde{B}_{\mu\nu}^{(g)} \right. \\ &\left. + \frac{i}{2} \tilde{\Phi}^{\mu(g)} \cdot \tilde{\Phi}_\mu^{(g)} + \tilde{F}^{(h)} \cdot \tilde{F}^{(h)} \right], \end{aligned} \quad (28)$$

where the expressions for superfields have been taken after the applications of HC and GIRs. It is now straightforward to note that the following mapping exists between the ordinary 4D symmetry and superfield formalism:

$$\frac{\partial}{\partial \bar{\theta}} [\tilde{\mathcal{L}}_B] = 0 \quad \Leftrightarrow \quad s_b \mathcal{L}_B = \partial_\mu [B \cdot D^\mu C]. \quad (29)$$

The above mapping is true due to the fact that (i) all the first three terms of the super Lagrangian density are effectively independent of the Grassmannian variables  $\theta$  and  $\bar{\theta}$ , (ii) there exists a relationship  $s_b \leftrightarrow \text{Lim}_{\theta \rightarrow 0} (\partial/\partial \bar{\theta})$ , and (iii) the translation operator  $(\partial/\partial \bar{\theta})$  along the  $\bar{\theta}$ -direction of the (4, 2)-dimensional supermanifold is nilpotent of order two (i.e.  $(\partial/\partial \bar{\theta})^2 = 0$ ).

As a consequence of the above continuous symmetry invariance, we can use the following Noether formula for the current in terms of the generic field  $\Psi_i = A_\mu, \Phi_\mu, B_{\mu\nu}, C, \bar{C}, B, \bar{B}$ , viz;

$$J_{(b)}^\mu = (s_b \Psi_i) \cdot \left( \frac{\partial \mathcal{L}_B}{\partial_\mu \Psi_i} \right) - B \cdot D^\mu C, \quad (30)$$

which leads to the derivation of conserved Noether current  $J_{(b)}^\mu$  as

$$\begin{aligned} J_{(b)}^\mu &= B \cdot D^\mu C - \left[ F^{\mu\nu} - \frac{m}{2} \varepsilon^{\mu\nu\eta\kappa} B_{\eta\kappa} \right] \cdot D_\nu C \\ &- \frac{m}{2} \varepsilon^{\mu\nu\eta\kappa} (\Phi_\nu \times C) \cdot B_{\eta\kappa} + \frac{i}{2} \partial^\mu \bar{C} \cdot (C \times C). \end{aligned} \quad (31)$$

The above expression can be recast in the following (more readable) form

$$\begin{aligned} J_{(b)}^\mu &= B \cdot D^\mu C - \partial^\mu B \cdot C - \frac{i}{2} (\partial^\mu \bar{C} \times C) \cdot C \\ &+ \partial_\nu \left[ (F^{\nu\mu} + \frac{m}{2} \varepsilon^{\mu\nu\eta\kappa} B_{\eta\kappa}) \cdot C \right], \end{aligned} \quad (32)$$

by exploiting the following set of Euler-Lagrange equations of motion

$$\begin{aligned} D_\mu F^{\mu\nu} + \frac{m}{2} \varepsilon^{\nu\mu\eta\kappa} \tilde{D}_\mu B_{\eta\kappa} - \partial^\nu B &= + i (\partial^\nu \bar{C} \times C), \\ \partial_\mu (D^\mu C) &= 0, \quad D_\mu (\partial^\mu \bar{C}) = 0, \quad F^{\mu\nu} = m \varepsilon^{\mu\nu\eta\kappa} B_{\eta\kappa}, \\ \varepsilon^{\mu\nu\eta\kappa} \tilde{D}_\nu B_{\eta\kappa} + 2 m \Phi^\mu &= 0, \quad \mathcal{F}_{\mu\nu} = 0, \end{aligned} \quad (33)$$

which emerge from the Lagrangian density  $\mathcal{L}_B$ . It should be recalled that we have taken  $\tilde{D}_\mu B_{\nu\eta} = \partial_\mu B_{\eta\kappa} - (A_\mu \times B_{\eta\kappa}) - (\Phi_\mu \times B_{\eta\kappa})$ . The conservation law  $\partial_\mu J_{(b)}^\mu = 0$  can also be proven by exploiting the above equations.

The conserved current  $J_{(b)}^\mu$  leads to the derivation of conserved charge  $Q_b = \int d^3x J_{(b)}^0$ . The explicit expression for this charge is

$$Q_b = \int d^3x \left[ B \cdot D^0 C - \dot{B} \cdot C - \frac{i}{2} \dot{\bar{C}} \cdot (C \times C) \right], \quad (34)$$

where we have dropped the total space derivative terms because of the Gauss divergence theorem. The above expression for the BRST charge is exactly same in appearance as the charge in the case of self-interacting non-Abelian 1-form gauge theory. However, there is a key difference because, in the above,  $\dot{B}$  has the explicit form (due to Euler-lagrange equation of motion) as

$$\dot{B} \equiv \partial^0 B = D_i F^{i0} + \frac{m}{2} \varepsilon^{0ijk} \tilde{D}_i B_{jk} - i (\dot{\bar{C}} \times C), \quad (35)$$

which contains the basic fields  $(A_\mu, \Phi_\mu, C, \bar{C})$  as well as auxiliary field  $B_{\mu\nu}$  (and their derivatives). Only in the limits  $B_{\mu\nu} \rightarrow 0, \Phi_\mu \rightarrow 0$  does the expression for the above charge reduces to the case of non-Abelian 1-form theory.

It is worthwhile to point out that the expression for  $Q_b$  contains the canonical momenta for all the dynamical fields of our present theory. However, as is evident, there is no momentum for the  $B_{\mu\nu}$  field in the expression for the

above charge. Thus, the following general formula for the BRST symmetry transformations (in terms of  $Q_b$ ), namely;

$$s_b \Phi_i = -i \left[ \Psi_i, Q_b \right]_{(\pm)}, \quad \Psi_i = A_0, A_i, C, \bar{C}, \Phi_0, \Phi_i, \quad (36)$$

where  $(\pm)$  signs on the square bracket correspond to the (anti)commutators for the generic field  $\Psi_i$  being (fermionic)bosonic in nature, is capable of producing the BRST transformations for all the *dynamical* fields. However, it is interesting to point out that it fails, for the obvious reasons, to generate the BRST transformation (i.e.  $s_b B_{\mu\nu} = -(B_{\mu\nu} \times C)$ ) for the field  $B_{\mu\nu}$ .

It is worthwhile to state that, besides  $B_{\mu\nu}$  field, there are other auxiliary fields in the theory. These are nothing but the Nakanishi-Lautrup (NL) type auxiliary fields. However, there is a distinct difference between the above cited auxiliary fields. As pointed out after equation (9), the requirements of the nilpotency and anticommutativity of the (anti-)BRST symmetry transformation  $s_{(a)b}$  produce the (anti-)BRST symmetry transformations for the NL type auxiliary fields  $B$  and  $\bar{B}$ . However, even these sacrosanct requirements of the BRST formalism do *not* produce the BRST transformation for the  $B_{\mu\nu}$  field. This is a novel feature of our present theory which has not been observed in the application of the BRST formalism to 4D (non-)Abelian 1-form and Abelian 2-form and 3-form gauge theories [21,16,17]. Thus, ultimately, we note that there is a key difference between the NL type auxiliary fields and the auxiliary field  $B_{\mu\nu}$  of our present theory.

## 5. Anti-BRST charge: as the symmetry generator

It can be checked from the anti-BRST symmetry transformations (listed in equations (9), (11), (17) and (22)) that the Lagrangian density  $\mathcal{L}_{\bar{B}}$  of (25), transforms to a total spacetime derivative (i.e.  $s_{ab}\mathcal{L}_{\bar{B}} = -\partial_\mu[\bar{B} \cdot D^\mu \bar{C}]$ ) under these transformations. This anti-BRST invariance can be written in terms of the superfields, obtained after the application of HC and GIRs, within the framework of the “augmented” BT superfield formalism. Towards this goal in mind, let us express the Lagrangian density  $\mathcal{L}_{\bar{B}}$  as

$$\begin{aligned} \tilde{\mathcal{L}}_B = & -\frac{1}{4} \tilde{F}^{\mu\nu(h)} \cdot \tilde{F}_{\mu\nu}^{(h)} + \frac{m^2}{2} \tilde{\Phi}^{\mu(g)} \cdot \tilde{\Phi}_\mu^{(g)} + \frac{m}{4} \varepsilon^{\mu\nu\eta\kappa} \tilde{\mathcal{F}}_{\mu\nu}^{(g,h)} \cdot \tilde{B}_{\eta\kappa}^{(g)} \\ & - \frac{\partial}{\partial\theta} \frac{\partial}{\partial\bar{\theta}} \left[ \frac{i}{2} \tilde{B}^{\mu(h)} \cdot \tilde{B}_\mu^{(h)} + \frac{1}{4} \tilde{B}^{\mu\nu(g)} \cdot \tilde{B}_{\mu\nu}^{(g)} \right. \\ & \left. + \frac{i}{2} \tilde{\Phi}^{\mu(g)} \cdot \tilde{\Phi}_\mu^{(g)} + \tilde{\bar{F}}^{(h)} \cdot \tilde{F}^{(h)} \right]. \end{aligned} \quad (37)$$

It is pretty obvious now to note the following mapping

$$\frac{\partial}{\partial \theta} [\tilde{\mathcal{L}}_B] = 0 \quad \Leftrightarrow \quad s_{ab} \mathcal{L}_{\bar{B}} = - \partial_\mu [\bar{B} \cdot D^\mu \bar{C}]. \quad (38)$$

The arguments for the above equivalence between the (4, 2)-dimensional superfield formalism and the continuous anti-BRST symmetry in the ordinary 4D space is exactly same as we have discussed in the previous section.

Exploiting the following Noether formula for the current in terms of the generic field  $\Psi_i = A_\mu, \Phi_\mu, B_{\mu\nu}, C, \bar{C}, B, \bar{B}$  of the Lagrangian density  $\mathcal{L}_{\bar{B}}$ :

$$J_{(ab)}^\mu = (s_{ab} \Psi_i) \cdot \left( \frac{\partial \mathcal{L}_B}{\partial_\mu \Psi_i} \right) + \bar{B} \cdot D^\mu \bar{C}, \quad (39)$$

we obtain the expression for the anti-BRST current  $J_{(ab)}^\mu$  as

$$\begin{aligned} J_{(ab)}^\mu &= -\bar{B} \cdot D^\mu \bar{C} - [F^{\mu\nu} - \frac{m}{2} \varepsilon^{\mu\nu\eta\kappa} B_{\eta\kappa}] \cdot D_\nu \bar{C} \\ &- \frac{m}{2} \varepsilon^{\mu\nu\eta\kappa} (\Phi_\nu \times \bar{C}) \cdot B_{\eta\kappa} + \frac{i}{2} \partial^\mu C \cdot (\bar{C} \times \bar{C}). \end{aligned} \quad (40)$$

Using the following Euler-Lagrange equations of motion

$$\begin{aligned} D_\mu F^{\mu\nu} + \frac{m}{2} \varepsilon^{\nu\mu\eta\kappa} \tilde{D}_\mu B_{\eta\kappa} + \partial^\nu \bar{B} &= -i (\bar{C} \times \partial^\nu C), \\ \partial_\mu (D^\mu \bar{C}) &= 0, \quad D_\mu (\partial^\mu C) = 0, \quad F^{\mu\nu} = m \varepsilon^{\mu\nu\eta\kappa} B_{\eta\kappa}, \\ \varepsilon^{\mu\nu\eta\kappa} \tilde{D}_\nu B_{\eta\kappa} + 2m \Phi^\mu &= 0, \quad \mathcal{F}_{\mu\nu} = 0, \end{aligned} \quad (41)$$

derived from the Lagrangian density  $\mathcal{L}_{\bar{B}}$ , one can, not only prove the conservation law  $\partial_\mu J_{(ab)}^\mu = 0$ , but also re-express the above conserved anti-BRST current in a compact form as given below

$$\begin{aligned} J_{(ab)}^\mu &= -\bar{B} \cdot D^\mu \bar{C} + \partial^\mu \bar{B} \cdot \bar{C} + \frac{i}{2} (\partial^\mu C \times \bar{C}) \cdot \bar{C} \\ &+ \partial_\nu \left[ (F^{\nu\mu} + \frac{m}{2} \varepsilon^{\mu\nu\eta\kappa} B_{\eta\kappa}) \cdot C \right], \end{aligned} \quad (42)$$

where we have exploited the Leibnitz rule of the operation of spacetime derivative on a set of combination of local fields.

The above conserved current leads to the definition of the conserved charge. This anti-BRST charge  $Q_{ab} = \int d^3x J_{(ab)}^0$ , is as follows:

$$Q_{ab} = - \int d^3x [\bar{B} \cdot D^0 \bar{C} - \dot{\bar{B}} \cdot \bar{C} - \frac{i}{2} \dot{C} \cdot (\bar{C} \times \bar{C})]. \quad (43)$$

From the definition of the canonical momenta (derived from the Lagrangian density  $\mathcal{L}_{\bar{B}}$ ), it can be checked that the above anti-BRST charge contains the momenta of all the fields except the auxiliary field  $B_{\mu\nu}$ . To see it clearly, we express here the time derivative on  $\bar{B}$  in terms of the other fields, namely;

$$\dot{\bar{B}} \equiv \partial^0 \bar{B} = -D_i F^{i0} - \frac{m}{2} \varepsilon^{0ijk} \tilde{D}_i B_{jk} - i (\bar{C} \times \dot{C}). \quad (44)$$

As a consequence, the conserved and nilpotent anti-BRST charge  $Q_{ab}$  generates all the anti-BRST symmetry transformations for all the fields *except*  $B_{\mu\nu}$ . As argued earlier, the auxiliary field  $B_{\mu\nu}$  is completely different from the Nakanishi-Lautrup type of auxiliary fields  $B$  and  $\bar{B}$  in the sense that we obtain the (anti-)BRST symmetry transformations for the latter fields by the requirements of nilpotency and anticommutativity of  $s_{(a)b}$  but we are unable to do so for the former auxiliary field. This is a novel observation in our present BRST analysis of the FT model for the non-Abelian TMGT.

## 6. Ghost charge and BRST algebra: a synopsis

Let us focus on the ghost part of the coupled (but equivalent) Lagrangian densities. It is clear that these terms (along with the total Lagrangian densities) are invariant under the following global scale transformations

$$C \rightarrow e^{+\Sigma}, \quad \bar{C} \rightarrow e^{-\Sigma}, \quad (A_\mu, \Phi_\mu, B_{\mu\nu}) \rightarrow (A_\mu, \Phi_\mu, B_{\mu\nu}), \quad (45)$$

where  $\Sigma$  is a global parameter and  $\pm$  signs, in the exponentials, denote the ghost number ( $\pm 1$ ) for  $C$  and  $\bar{C}$ , respectively. The above transformations also show that the ghost number for the fields  $(A_\mu, \Phi_\mu, B_{\mu\nu})$  is zero. The infinitesimal version of the above scale transformations leads to the derivation of conserved current  $J_{(g)}^\mu$  and corresponding charge  $Q_g = \int d^3x J_{(g)}^0$  as<sup>9</sup>

$$J_{(g)}^\mu = i [\bar{C} \cdot D^\mu C - \partial^\mu \bar{C} \cdot C], \quad Q_g = i \int d^3x [\bar{C} \cdot D^0 C - \dot{\bar{C}} \cdot C]. \quad (46)$$

By exploiting the equations of motion ( $\partial_\mu D^\mu C = 0, D_\mu \partial^\mu \bar{C} = 0$ ), derived from the Lagrangian density  $\mathcal{L}_B$ , it can be checked that the above charge and current are conserved and  $Q_g$  turns out to be the generator of the infinitesimal version of the scale transformations listed above (cf. (45)).

We can tap the potential and power of the idea of a generator to derive the BRST algebra amongst the conserved charges  $Q_b, Q_{ab}, Q_g$ . For instance,

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<sup>9</sup>We have taken the Lagrangian density  $\mathcal{L}_B$  for the computation of the ghost current and corresponding charge. However, one can choose the other Lagrangian density  $\mathcal{L}_{\bar{B}}$  as well for this kind of computation to obtain the expressions for the current and charge.



we can check that  $s_b Q_b == -i \{Q_b, Q_b\} = 0$ ,  $s_b Q_{ab} = -i \{Q_{ab}, Q_b\} = -i (Q_b Q_{ab} + Q_{ab} Q_b) = 0$ , etc. Collecting all such computations, we find that the following standard BRST algebra

$$\begin{aligned} Q_b^2 &= 0, & Q_{ab}^2 &= 0, & Q_b Q_{ab} + Q_{ab} Q_b &\equiv \{Q_b, Q_{ab}\} = 0, \\ i [Q_g, Q_b] &= +Q_b, & i [Q_g, Q_{ab}] &= -Q_{ab}, & i [Q_g, Q_b Q_{ab}] &= 0, \end{aligned} \quad (47)$$

emerges from the above conserved charges. It is worth pointing out that

(i) the absolute anticommutativity, between  $Q_b$  and  $Q_{ab}$ , requires the validity of CF condition, and

(ii) from the above algebra, it is clear that the ghost numbers for the charges  $Q_b$ ,  $Q_{ab}$  and  $Q_b Q_{ab}$  are  $+1$ ,  $-1$  and  $0$ , respectively.

## 7. Conclusions

We have accomplished our central goal of obtaining the off-shell nilpotent and absolutely anticommuting (anti-)BRST transformations  $s_{(a)b}$  corresponding to the (1-form) YM gauge symmetry transformations (2) for the original FT Lagrangian density  $\mathcal{L}_0$  of equation (1). In this attempt, the “augmented” BT superfield formalism has played a key role as it has led to the derivation of CF condition (cf. equation (7)) that is responsible for the absolute anticommutativity of  $s_{(a)b}$  and the derivation of the coupled (but equivalent) Lagrangian densities (cf. equation (25)). In this context, a novel observation is the fact that one is theoretically compelled to invoke GIRs, in addition to the celebrated HC, in the application of BT superfield formalism to the description of the 4D topologically massive non-Abelian gauge theory. To be specific, we obtain the (anti-)BRST transformations for the  $B_{\mu\nu}$  and  $\Phi_\mu$  fields only when HC and GIRs blend together in a meaningful manner<sup>10</sup>.

One of the main motivations for our present investigation was to conduct a comparative study of the FT model within the framework of the BRST and superfield formulations *vis-à-vis* such a kind of study performed for the dynamical non-Abelian 2-form (topologically massive) gauge theory, described by the following Lagrangian density with the topological  $(B \wedge F)$  term [6,7,9]

$$\mathcal{L}_D = -\frac{1}{4} F^{\mu\nu} \cdot F_{\mu\nu} + \frac{1}{12} H^{\mu\nu\eta} \cdot H_{\mu\nu\eta} + \frac{m}{4} \varepsilon^{\mu\nu\eta\kappa} B_{\mu\nu} \cdot F_{\eta\kappa}, \quad (48)$$

where the 3-form  $(H^{(3)} = \frac{1}{3!} (dx^\mu \wedge dx^\nu \wedge dx^\eta) H_{\mu\nu\eta})$  curvature tensor  $H_{\mu\nu\eta}$  is defined in terms of the 1-form gauge field  $A_\mu$ , 1-form auxiliary field  $K_\mu$  and

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<sup>10</sup>We christen this modified version of the BT superfield approach (where HC and GIRs must blend together for proper application) as the “augmented” BT superfield formalism.

the dynamical 2-form ( $B^{(2)} = \frac{1}{2!}(dx^\mu \wedge dx^\nu)B_{\mu\nu}$ ) gauge field  $B_{\mu\nu}$  as

$$H_{\mu\nu\eta} = D_\mu B_{\nu\eta} + D_\nu B_{\eta\mu} + D_\eta B_{\mu\nu} - (K_\mu \times F_{\nu\eta} + K_\nu \times F_{\eta\mu} + K_\eta \times F_{\mu\nu}). \quad (49)$$

In the above, we have taken the usual definition of the covariant derivative as:  $D_\mu B_{\nu\eta} = \partial_\mu B_{\nu\eta} - (A_\mu \times B_{\nu\eta})$ . It was observed, in the BRST analysis (corresponding to the 1-form YM gauge symmetries) of this model, that the conserved and nilpotent (anti-)BRST charges were unable to generate the (anti-)BRST symmetry transformations for the component  $B_{0i}$  of the 2-form dynamical field  $B_{\mu\nu}$  as well as the auxiliary 1-form field  $K_\mu$ , even though, the transformations of these fields were taken into account in the derivation of the above charges [20]. In contrast, we have shown, in our present study, that the (anti-)BRST charges of the FT model, are *not* capable of generating the nilpotent (anti-)BRST symmetry transformations corresponding to the 2-form auxiliary field  $B_{\mu\nu}$  of our present non-Abelian version of TMGT.

We find that there is a distinct difference between the Nakanishi-Lautrup type auxiliary fields  $B, \bar{B}$  and the auxiliary field  $B_{\mu\nu}$ . As is evident from our present discussions, the requirements of the off-shell nilpotency and absolute anticommutativity of the (anti-)BRST symmetry transformations lead to the determination of these symmetry transformations for  $B$  and  $\bar{B}$ . On the contrary, the above requirements are found to be *not* good enough to produce the (anti-)BRST symmetry transformations for the field  $B_{\mu\nu}$ . The central reason for this discrepancy is the fact that the momentum, for the field  $B_{\mu\nu}$ , does not appear in the expressions for the (anti-)BRST charges, even though, we perform the BRST analysis in its most general form (cf. equation (24)). In fact, the gauge-fixing and Faddeev-Popov ghost terms do *not* appear for the fields  $B_{\mu\nu}$  and  $\Phi_\mu$  in the most general Lagrangian densities where the basic tenets of BRST formalism have been exploited in their full generality.

The present model has also been discussed within the framework of the superfield and BRST formalism in [24] where a tower of auxiliary fields have been invoked for the consistency of the BRST symmetries, equations of motion and integrability of the BRST equation. However, in this attempt, the CF condition does not appear which is the root cause for the absolute anticommutativity of the (anti-)BRST symmetry transformations. One of the key ingredients of the BT superfield approach to BRST formalism (that is applied to the description of any arbitrary  $p$ -form gauge theory) is the very natural derivation of the CF condition. We claim that the appearance of the latter condition (i.e. CF condition) is the hallmark of any  $p$ -form gauge theory described within the framework of BRST formalism.

We have established a deep connection of the CF type restrictions with geometrical objects called gerbes in a couple of papers where we have discussed

the Abelian 2-form and 3-form gauge theories [16,17]. We plan to establish connections of the CF type restrictions, appearing in the discussion of the non-Abelian TMGTs, with the concept of gerbes. Furthermore, it would be nice endeavor to exploit the tensor gauge symmetries of our present model within the framework of BRST and superfield formalisms. These are some of the current issues that are being pursued at the moment and our results would be reported in our future publications [25].

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